1. The table below shows the least costs, in pounds, of travelling between six cities, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	36	18	28	24	22
B	36	–	54	22	20	27
C	18	54	–	42	27	24
D	28	22	42	–	20	30
E	24	20	27	20	–	13
F	22	27	24	30	13	–

Vicky must visit each city at least once. She will start and finish at A and wishes to minimise the total cost.

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network.

(2)

(b) Use your answer to part (a) to help you calculate an initial upper bound for the length of Vicky's route.

(1)

(c) Show that there are two nearest neighbour routes that start from A. You must make your routes and their lengths clear.

(3)

(d) State the best upper bound from your answers to (b) and (c).

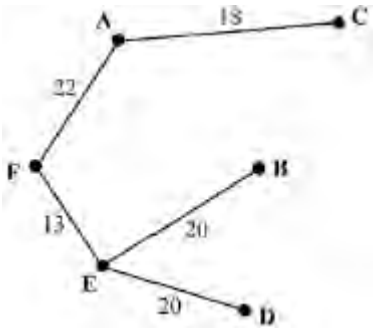
(1)

(e) Starting by deleting A, and all of its arcs, find a lower bound for the route length.

(4)

(Total 11 marks)

1. (a)



M1 A1 2

Notes

1M1: Spanning tree found. Allow 1×2×43 across top of table or 93
1A1: CAO must see tree or list of arcs

(b) Minimum Spanning tree length 93, so upper bound is £186 B1ft 1

Note

1B1ft: 186 their ft93 × 2

(c)

A C F E B D A	M1	
18 24 13 20 22 28 Length 125	A1	
A C F E D B A		
18 24 13 20 22 36 Length 133	A1	3

Notes

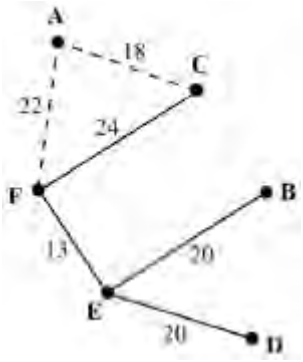
1M1: One Nearest Neighbour each vertex visited at least once (condone lack of return to start)
1A1: One correct route and length CAO – must return to start.
2A1: Second correct route and length CAO – must return to start.

(d) Best upper bound is £125 B1ft 1

Note

1B1ft: ft but only on three different values.

(e) Delete A



M1 A1

RMST weight = 77
Lower bound = 77 + 18 + 22 = £117 M1 A1 4

Notes

1M1: Finding correct RMST (maybe implicit) 77 sufficient,
or correct numbers. 4 arcs.
1A1: CAO tree or 77.
2M1: Adding 2 least arcs to A, 18 and 22 or 40 only
2A1: CAO 117

[11]

1. This proved a good first question with over a third of the candidates scoring full marks and over 50% getting at least 10 marks.

In part (a) many candidates did not list the arcs or draw the tree, BD was often included. Almost all were able to correctly find an initial upper bound based on their tree.

In (c) most candidates were able to find the two nearest neighbour routes but a surprisingly large number did not return to A, others found the NN route from A to B and A to D and then, alarmingly, doubled it.

Those who completed (c) correctly usually completed (d) correctly.

Many completed part (e) correctly but BD was, once again, often included in the residual minimum spanning tree.